Triangle Challenge

D version by Stewart Gordon

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triangles2.exe   Second console version (executable)

triangles3.exe   GUI version (executable)

Makefile   Makefile

Implementation notes

This solution to the Triangle Challenge is written in D <http://www.digitalmars.com/d/>.

The first console version merely calculates the number of triangles. The second console version displays the enumeration of triangles, as a rectangular grid that reduces to the triangular grid under the transformation described in this document.

For the GUI version, I have used SDWF <http://smjg.port5.com/pr/d/sdwf/>, a Windows programming library that I am developing. It is written under SDWF 0.4, which is likely to be released in early November. I believe it will compile under SDWF 0.3, although I haven't had time to check.

Determining the number

The problem is to calculate the number of triangles in this figure:
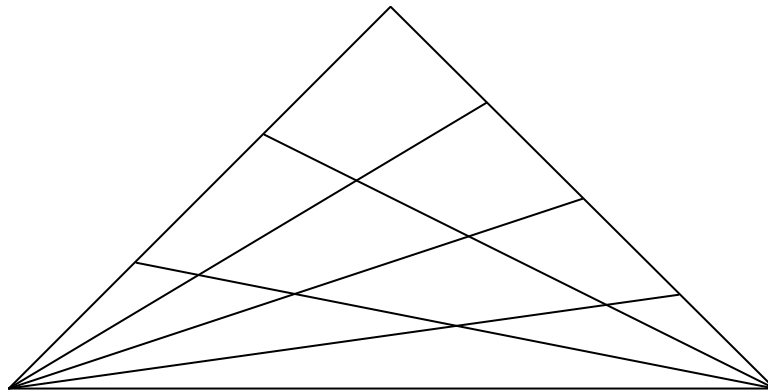
To solve the problem, we consider a rectangular version:
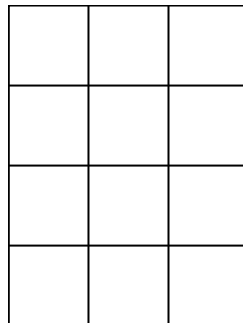
The number of rectangles in this figure is determined by the number of distinct ranges of rows and columns. The number of row ranges is determined by the triangle of the total number of rows in the grid:
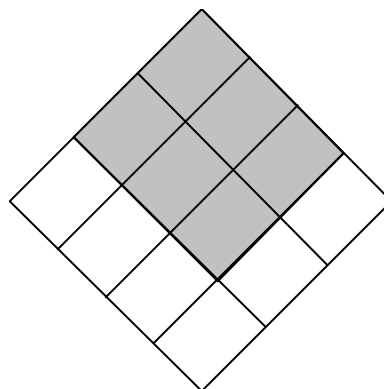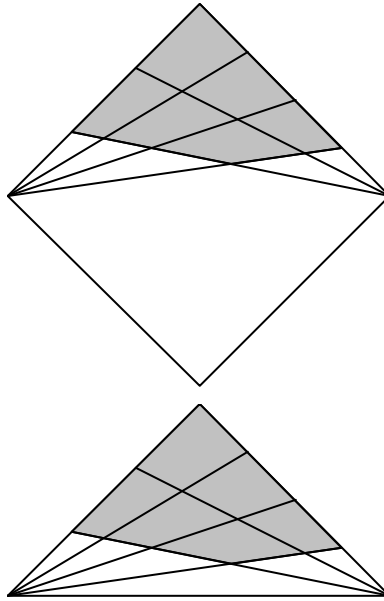
$$R_1 = \tfrac{1}{2} n_1 (n_1 + 1)$$

When there are two dimensions to consider, the set of rectangles is the Cartesian product of the set of rows and the set of columns:

$$R = \tfrac{1}{4} n_1 n_2 (n_1 + 1)(n_2 + 1)$$

where n_1 is the number of rows, and n_2 is the number of columns.

The rectangular grid reduces to a triangular grid by this sequence:

Of the rectangles in the rectangular grid, only those that touch at least one of the two lower corners become triangles in the triangular grid. Consequently, all rectangles in the shaded areas remain quadrilaterals, and so must be excluded when calculating the number of triangles. The number of triangles is therefore determined:

$$T = \tfrac{1}{4} n_1 n_2 (n_1 + 1)(n_2 + 1) - \tfrac{1}{4}(n_1 - 1)(n_2 - 1)n_1 n_2$$
$$= \tfrac{1}{2} n_1 n_2 (n_1 + n_2)$$

Enumerating them

Calculating the number is one thing, but actually listing them all is quite another.

There are three kinds of triangles in the figure:

Touching lower vertices	Number
Left only	$\tfrac{1}{2} n_1 n_2 (n_1 - 1)$
Right only	$\tfrac{1}{2} n_1 n_2 (n_2 - 1)$
Both	$n_1 n_2$

Then a simple order of enumeration can be done. For the left-only triangles, iterate over the $\tfrac{1}{2}(n_1 - 1)n_1$ possible ranges of angles at the left vertex, and the n_2 triangle lengths. Do the right-only triangles similarly. For triangles that include the base side, the process is simpler – just iterate over the ranges up each lower vertex.

Geometry

(0, 1)

(−1, 0)

(1, 0)
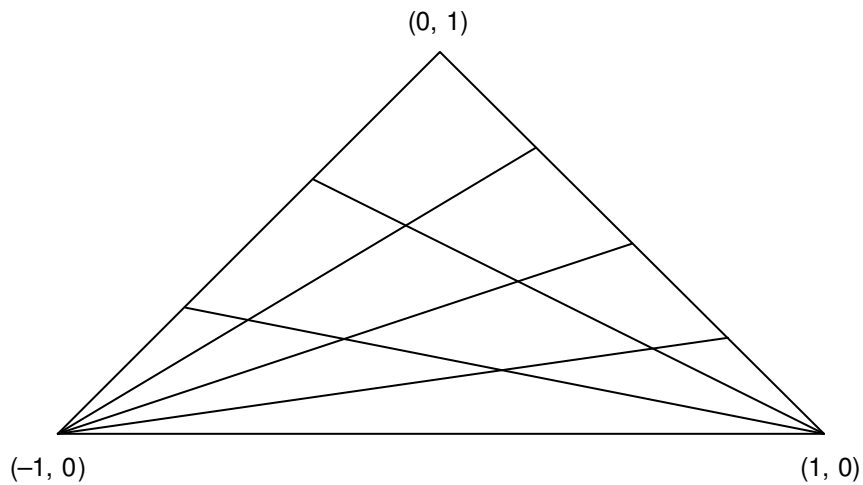
The lines from the left-hand lower vertex have the coordinates

$$\left(-1,\, 0\right) \rightarrow \left(1 - \frac{i_1}{n_1},\, \frac{i_1}{n_1}\right)$$

hence an equation of

$$i_1 x + \left(i_1 - 2n_1\right)y = -i_1$$

The lines from the right-hand lower vertex have the coordinates

$$\left(1,\, 0\right) \rightarrow \left(\frac{i_2}{n_2} - 1,\, \frac{i_2}{n_2}\right)$$

giving the equation

$$i_2 x + \left(2n_2 - i_2\right)y = i_2$$

Solving these equations gives

$$x = \frac{i_1 n_2 - n_1 i_2}{i_1 i_2 - n_1 i_2 - i_1 n_2}$$

$$y = -\frac{i_1 i_2}{i_1 i_2 - n_1 i_2 - i_1 n_2}$$

Progress

Task	Time taken (min) (approx)
Theory: determining the number	30
Coding: first console version	10
Theory: enumerating them	15
Coding: second console version	35
Theory: geometry	35
Coding: GUI version	60